For Ch. 9 be able to

1. Solve separable diff. eq.
2. Use initial conditions & constants.
3. Set up and do ALL the applied problems from homework.

Worried about applied problems?

Pay attention today and next lecture. Know the homework well. And go thru my review sheets and look at old finals.

9.4 Differential Equations Applications

Newton's Cooling Law Experiment

Hot water is in the cup. We will try to predict the temp. at the end of class.

1st measurement:

Time = Temp =

2nd measurement:

Time = Temp =

***1. Law of Natural Growth/Decay*:**

Assumption: "*The rate of growth/decay is proportional to the function value*."

$$\frac{dP}{dt} = kP \text{ with } P(0) = P_0$$

Example:

The *half-life* of cesium-137 is 30 years. Suppose we start with a 100-mg sample. Find m(t).

Example:

Bob deposits \$2000 into a savings account. The money grows at a rate proportional to its size (*i.e.* compound interest). The balance in 4 years is \$2100. Find the formula $A(t)$ for the amount in his account in t years.

2. Newton's Law of Cooling:

Assumption: "*The rate of temperature change is proportional to the difference between the temperature of the object and its surroundings*."

3. Mixing Problems:

Assume a vat of water has a *contaminant* entering at some rate and exiting at some rate, then "*The rate of change of the contaminant is equal to the rate at which the contaminant is coming IN minus the rate at which it is going OUT.*"

Assume a 12 Liter vat contains 7 kg of salt initially. A pipe pumps in *pure water* at 3 L/min.
The vat is well mixed. The mixture drains at 3 L/min.
Let $y(t)$ = "kg of salt in vat at time t".

Identify and label the following:
1. Volume of the vat (Is it changing?)
2. Amount of salt per min entering.
3. Amount of salt per min exiting.
4. Initial amount of salt.

Example: Assume a 12 Liter vat contains 7 kg of salt initially. A pipe pumps in salt water (brine) at 3 L/min with a concentration of 2 kg/L of salt. The vat is well mixed. The mixture drains at 3 L/min.

Let $y(t)$ = "kg of salt in vat at time t".

(a) Find $y(t)$.

(b) Find the limit of $y(t)$ as $n \to \infty$.

Mixing Problem Summary

V = volume of vat (liters)

t = time (min)

$y(t)$ = amount in vat (kg)

$\frac{dy}{dt}$ = rate (kg/min)

$$\frac{dy}{dt} = \text{Rate In } - \text{Rate out}$$

$$= \left(?\frac{kg}{L}\right)\left(?\frac{L}{min}\right) - \left(\frac{y}{V}\ \frac{kg}{L}\right)\left(?\frac{L}{min}\right)$$

$$y(0) = ?\ \text{kg}$$

Example: Assume a 100 Liter vat contains 5kg of salt initially. Two pipes (A & B) pump in salt water (brine).

Pipe A: Enters at 3L/min with a concentration of 4kg/L of salt.

Pipe B: Enters at 5L/min with a concentration of 2kg/L of salt.

The vat is well mixed.

The mixture leaves the vat at 8L/min.

Let $y(t)$ = “kg of salt in vat at time t”.

How would you set this up?

Example: Assume a 50 Liter container currently has 20 Liters of water with 24 kg of dissolved salt.
A pipe pumps in *pure water* at 6 L/min.
The vat is well mixed.
The mixture drains at 4 L/min.
Let $y(t)$ = "kg of salt in vat at time t".
What is different about this problem?

4. *Air Resistance:*

A skydiver steps out of a plane that is 4,000 meters high with and initial downward velocity of 0 m/s. The skydiver has a mass of 60 kg. (Treat downward as positive).

Let y(t) = "height at time *t*"
Let v(t) = y'(t) = "velocity at time *t*"
Let a(t)=v'(t)='''(t) = "accel. at time *t*"

Newton's 2nd Law says:

(mass)(acceleration) = Force

$$m\frac{d^2y}{dt^2} = \text{sum of forces on the object}$$

The force due to gravity has constant magnitude (and it is acting downward):
$F_g = mg = 60 \cdot 9.8 = 588$ N

One model for air resistance

The force due to air resistance (*drag force*) is proportional to velocity and in the opposite direction of velocity. So

$$F_d = -k\,v \quad \text{Newtons}$$

Assume for this problem k = 12.

The Logistics Equation

Consider a population scenario where there is a limit to the amount of growth (spread of a rumor, for example).

Let $P(t)$ = population size at time t.

M = maximum population size. (capacity)

We want a model that

…is like natural growth when P(t) is significantly smaller than M;

…levels off (with a slope approaching zero), then the population approaches M.

One such model is the so-called logistics equation

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right) \text{ with } P(0) = P_0$$

Random old final questions:

Spring 2011 Final:

Brief summary of what it says:

$v(t)$ = velocity of an object

$$F = mg - kv$$

Recall:

$$F = ma = m\frac{dv}{dt}$$

You are given m, g, and k and asked for solve for $v(t)$.

Spring 2014:

A lake has a volume of 1000 cubic meters and contains pure water. A farm near the lake begins using a pesticide. Runoff from the farmland into the lake is 10 cubic meters of water per day, with a concentration of 50 grams of pesticide per cubic meter of water. The lake drains to the ocean at a rate of 10 cubic meters per day.

Winter 2011

Your friend wins the lottery, and gives you P_0 dollars to help you pay your college expenses. The money is invested in a savings account that earns 10% annual interest, compounded continuously, and you withdraw the money continuously (a pretty good approximation to reality if you make regular frequent withdrawals) at a rate of $3600 per year.

Fall 2009

The swine flu epidemic has been modeled by the Gompertz function, which is a solution of

$$\frac{dy}{dt} = 1.2\, y\, (K - ln(y)),$$

where $y(t)$ is the number of individuals (in thousands) in a large city that have been infected by time t, and K is a constant.

Time t is measured in months, with t = 0 on July 9, 2009.

On July 9, 2009, 75 thousand individuals had been infected.

One month later, 190 thousand individuals had been infected.

1. 500 bacteria are in a dish at t=0hr.
8000 bacteria are in the dish at t=3hr.
Assume the population grows at a rate proportional to its size.
Find the function, B(t), for the bacteria population with respect to time.

2. The *half-life* of cesium-137 is 30 years. Suppose we start with a 100-mg sample. The mass decays at a rate proportional to its size.
Find the function, m(t), for the mass with respect to time.

3. You invest $10,000 into a savings account and never make any deposits or withdrawals. The balance grows at a rate proportional to its size (i.e. *interest* is a percentage of the balance at any time). In 3 years, you notice your balance is $10,400.

Find the function, A(t), for the amount of money in the account with respect to time.